From Risk Analysis to Adversarial Risk Analysis

Part I. Problem Structuring. IDs

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Review of basic concepts

- Decision Analysis cycle
- Influence diagrams
- Belief modelling
- Risk attitudes
- Bayesian computational methods
- Mixture modelling

Decision Analysis

- Purpose

Support a Decision Maker in making a decision under uncertainty (consequences of decisions not known with certainty).

A prescriptive approach.

(Normative, Descriptive)

Decision analysis (cycle)

- Structure problem: Identify alternatives, uncertainties and consequences
- Elicit probabilities, Possibly update in light of data
- Elicit utilities
- Compute alternative with maximum (posterior) expected utility
- Perform sensitivity analysis
- Possibly iterate, until implementation

$$\max_a \int u(c(a,\theta))p(\theta|x)d\theta$$

Three problem structuring tools

- Decision Tables
- Decision Trees
- Influence Diagrams

- Investment decision problem
  - One **decision** variable with two alternatives
    - Where to invest?
      - Treasury bonds
      - IBM shares
  - One **uncertainty** with two possible states
    - IBM share price at the end of the year
      - High
      - Low
  - One evaluation criteria for **consequences**
    - Profit from investment

Decision Table

In what to investment?

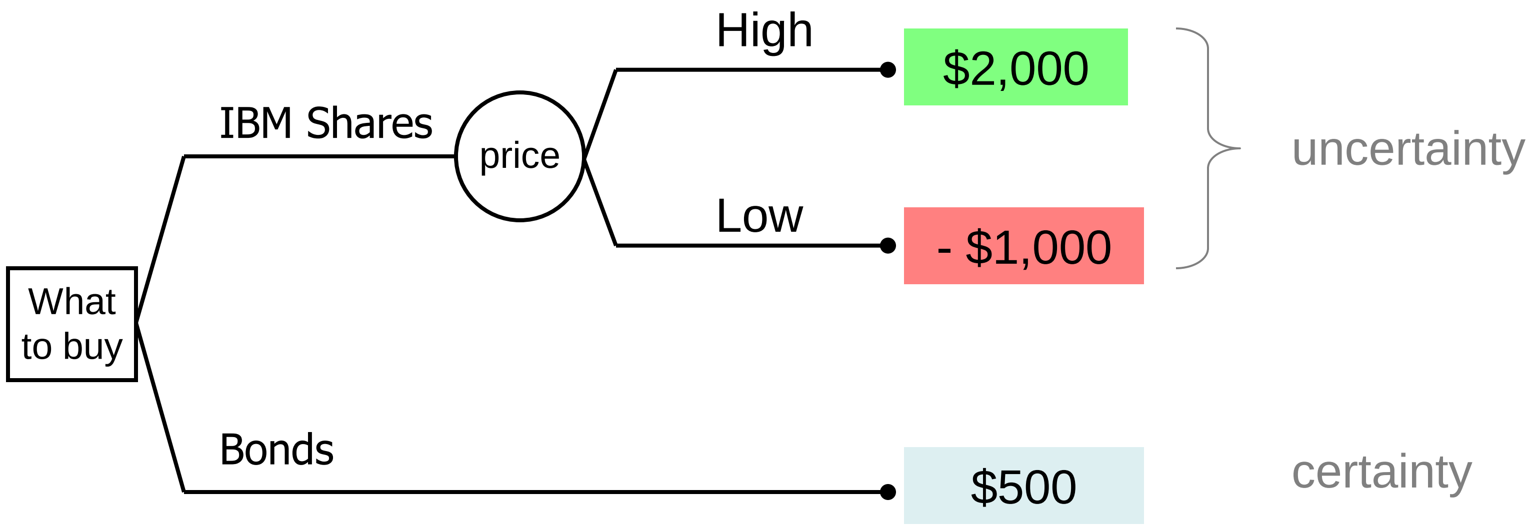
	IBM share price	
	High	Low
Bonds	$500	$500
Shares	$2,000	- $1,000

- DM chooses a row without knowing which column will occur
- Choice depends on the relative likelihood of High and Low?
  - If DM is sure that IBM share price will be **High**, best choice is to buy **Shares**
  - If DM is sure that IBM share price will be **Low**, best choice is to buy **Bonds**

  Elicit the DM's beliefs about which column will occur
- Choice depends on the value of money
  - Expected return not a good measure of decision preferences
    - The two alternatives give the same expected return but most of DMs would not fell indifferent between them

  Elicit risk attitude of the DM

Decision tree representation

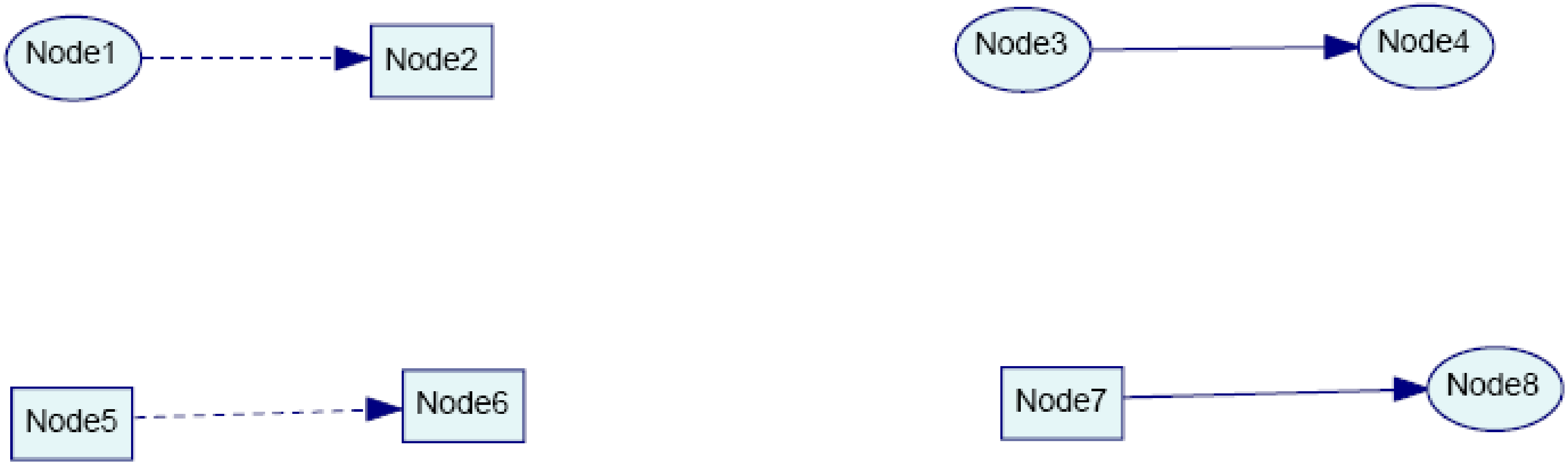


- What does the choice depends upon?
  - relative likelihood of H vs L
  - strength of preferences for money

Influence Diagrams

- Tool to structure (and solve) decision making problems
- Direct acyclic graph G=(N,A)
- Three main types of nodes.
  - Chance. Circle
  - Decision. Square
  - Value. Hexagon, Diamond
    - Fourth type of node. Deterministic. Double circle
- Two types of arcs
  - Arcs into decision nodes
  - Arcs into chance and value nodes

Influence Diagrams. Interpretation?



Influence Diagrams

- The hidden information

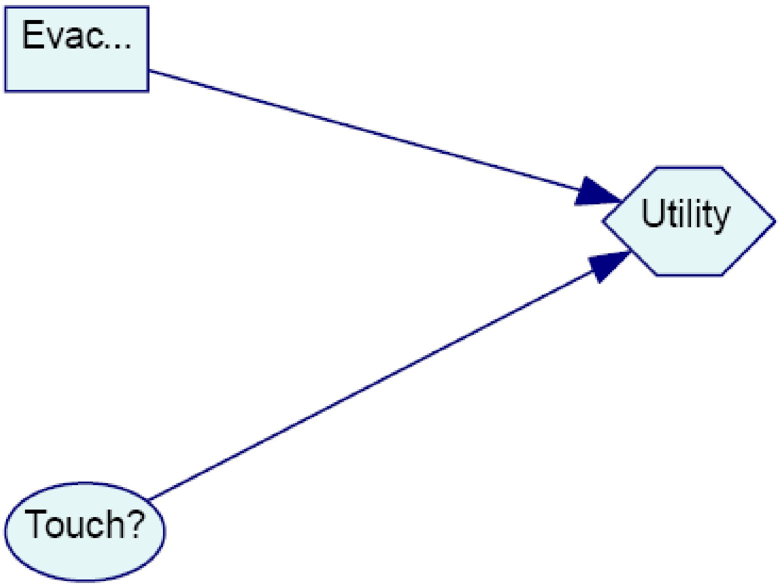
- Well-definedness

Acyclic, 1 value node, Memory

Structure IBM problem as influence diagram

Suppose you’re Miami’s mayor. There is a hurricane threat. Would you issue an evacuation order?

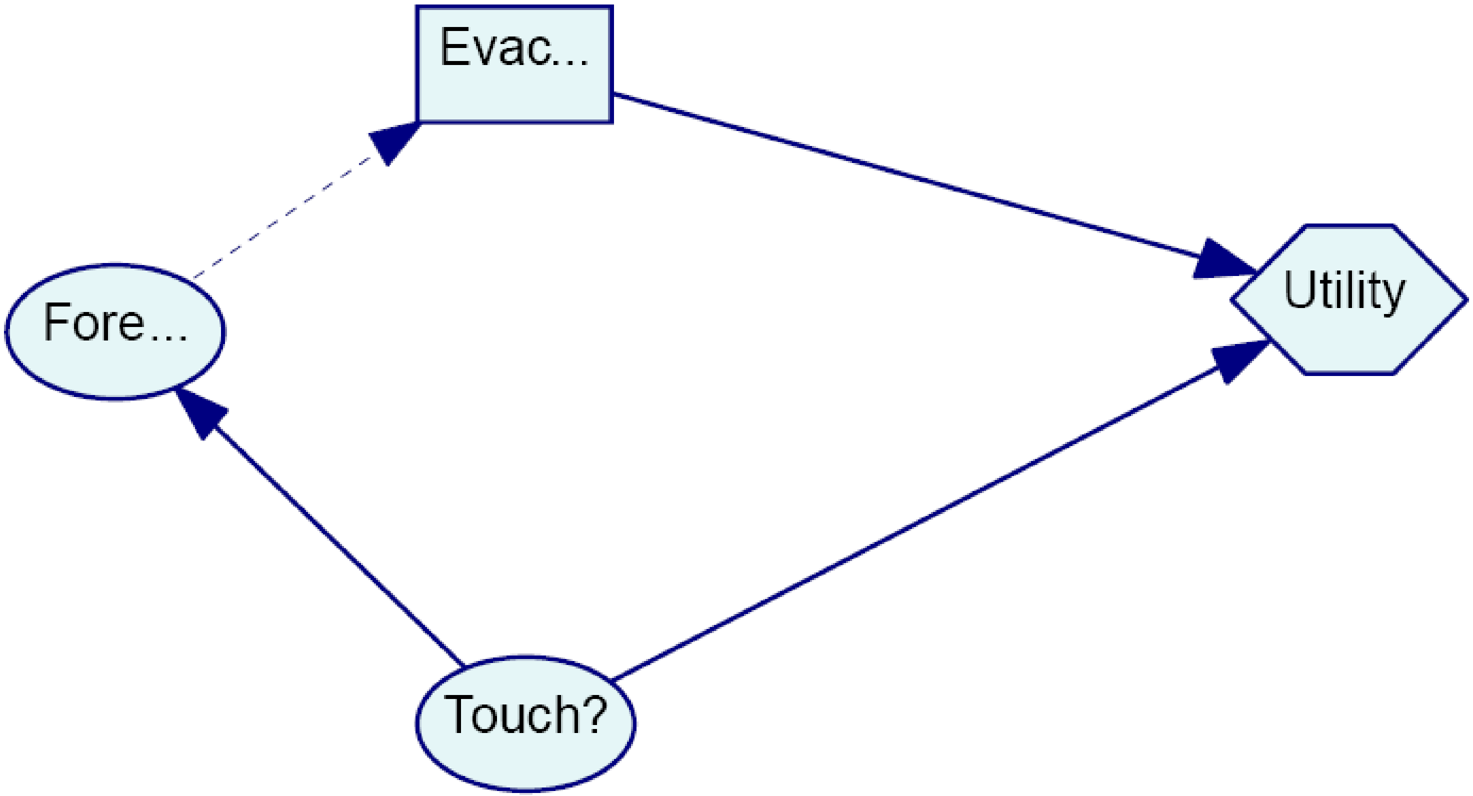
Decision under risk



Suppose you're Miami's mayor. There is a hurricane threat. Would you issue an evacuation order?

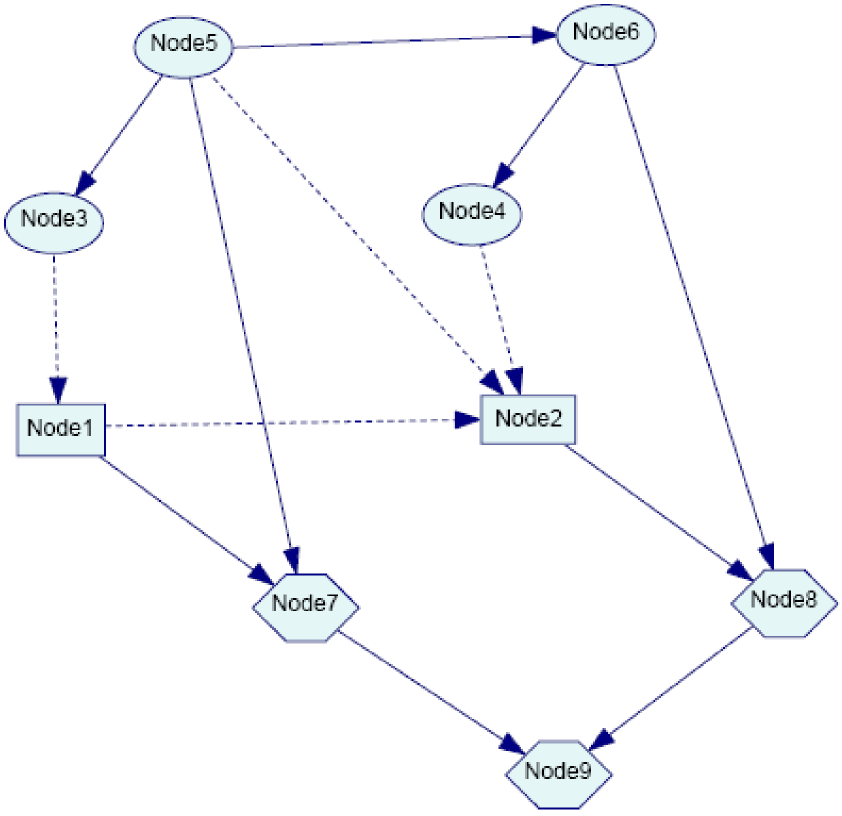
You have as info a forecast from the NCH. But the forecast is not perfect…

Decision under risk with imperfect information

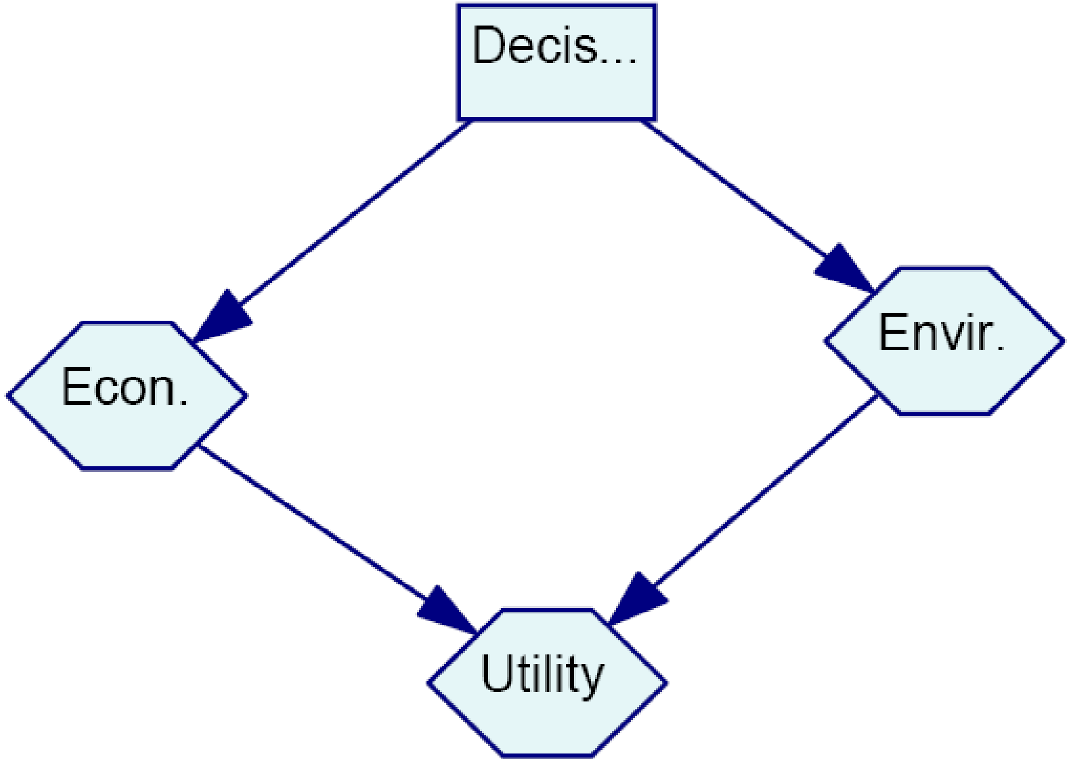


… the problem repeated over the hurricane season….

Sequential Decisions



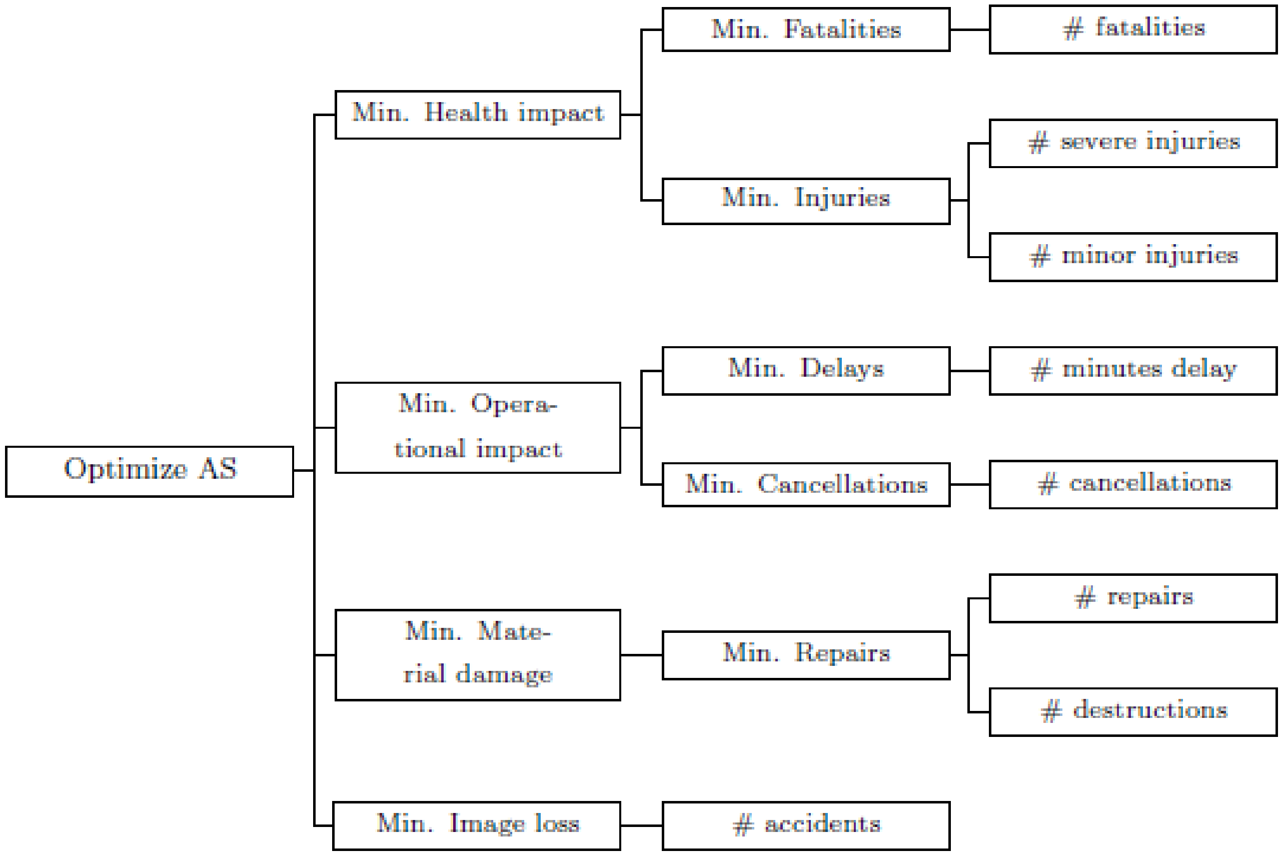
Influence diagrams: Multiple objectives



Multiple objectives

- Hierarchy
- From top level generic objective
- To more precise subobjectives
- Until low subobjectives determined.
  - Relevant
  - Non redundant
  - Measurable
- Natural, Constructed, Proxy

Multiple objectives in aviation safety. AESA



Handling uncertainty

We shall deal with issues in relation with modelling uncertainty. We assume a certain background typical of courses in Statistics. We shall focus on topics relevant for Decision Analysis, adopted from Bayesian Inference.

An intro is in

http://en.wikipedia.org/wiki/Bayesian_inference

Uncertainty

- Uncertainty is the lack of knowledge of what is or will happen. It is almost ubiquitous in our lives. Consider these statements:
  - Smoking causes cancer
  - Madrid will organise the Olympic Games in 2036
  - Mexico became independent in 1826
  - My weight is greater than 90 kgs
  - Hannibal crossed the Alps through San Bernardo
- Due to uncertainty, you do not know the consequences of your decisions before making them. Consider these examples:
  - The price of a Brent barrel will exceed 60 dollars by the end of year 2018
  - General elections will be repeated in Spain
  - Water demand in Madrid by 2030 will be higher than…
  - No storm is expected around the coast of Ferrol in the next two days.

Uncertainty

- We shall use probabilities to quantify uncertainty. The three main interpretations are:
  - *Classical*. With equally likely cases, the probability of an event is defined through

    Favourable cases/Possible cases

    http://en.wikipedia.org/wiki/Classical_definition_of_probability
  - *Frequentist*. If we may repeat indefinitely an experiment under identical conditions, the probability of an event is

    Limit of relative frequency of event

    http://en.wikipedia.org/wiki/Frequency_probability

    Operationally, relative frequency in a large number of trials

Uncertainty

– ***Subjective.*** The probability of an event is

A measure of the degree of belief in the occurrence of the event

http://en.wikipedia.org/wiki/Bayesian_probability

The most general and useful concept in Decision Analysis (and Risk Analysis)

Uncertainty

In general, when modeling uncertainty in a decision making problem, we need to deal with these issues:

- Which are the key uncertainties?
- Which are the possible outcomes of such uncertainties?
- Which are the probabilities of various outcomes?
- Which are the consequences entailed by such outcomes (for various alternatives)?

We deal with them in the next slides

Probabilistic diagrams

- As basic tools for qualitative modelling of uncertainty use probabilistic influence diagrams a.k.a. causal networks, Bayesian networks, Belief networks,…. See the excellent

http://en.wikipedia.org/wiki/Bayesian_network

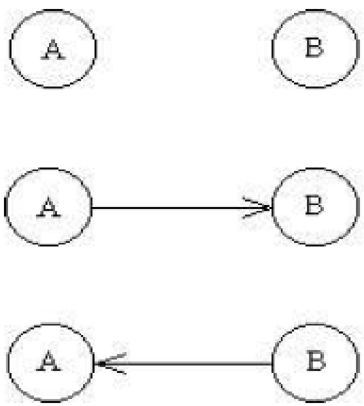
They are **influence diagrams** with chance nodes only. Qualitatively they describe a probabilistic model through

$$P(A1, A2,\ldots, An) = P(A1 \mid ant(A1))\ldots P(An \mid ant(An))$$

where ant (Ai) are the antecessors of node Ai.

In what follows we see several PIDs and we need to indicate the entailed probabilistic model

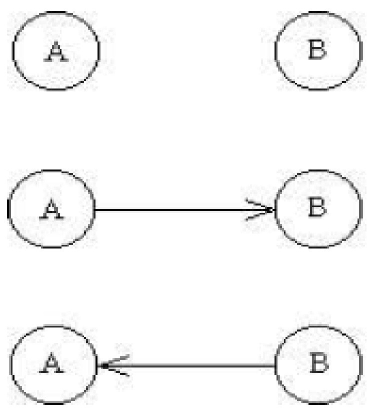
Probabilistic diagrams with two nodes



Before moving foreward, write the entailed probabilistic model

Probabilistic diagrams with two nodes

Model for P(A,B)



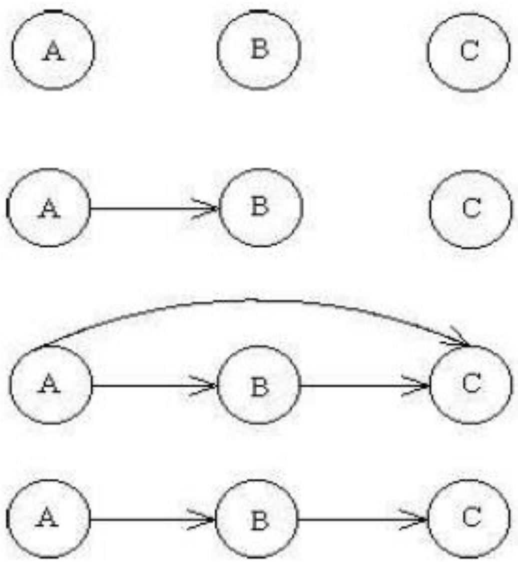
P(A)P(B)

P(A) P(B|A)

P(B) P(A|B)

First case, A and B are independent. We move from second to third, and viceversa, via Bayes formula

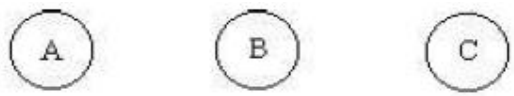
Probabilistic diagrams with three nodes



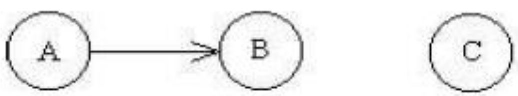
Before moving foreward, write the entailed probabilistic model

Probabilistic diagrams with three nodes

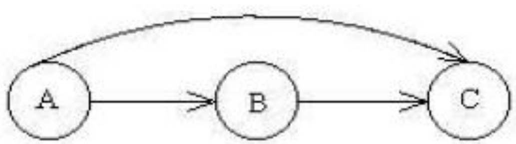
Model P(A ,B ,C)



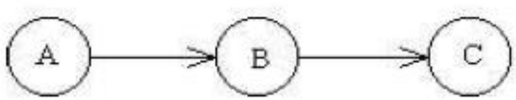
P(A)P(B)P(C)



P(A) P(B|A) P(C)



P(A)P(B|A)P(C|A,B)



P(A)P(B|A)P(C|B)

First case, independence. Third case, A and C are conditionally independent given B.

Read http://en.wikipedia.org/wiki/Conditional_independence

Probabilistic diagrams. Asia

An example referring to lung diseases, from Lauritzen and Spiegelhalter (1988):

A breathing condition (dyspnea) may be due to tuberculosis, lung cancer or bronchitis, none of them or several of them. A recent visit to Asia, increase the chances of tuberculosis, whereas smoking is a risk factor for lung cancer and bronchitis. The results of an X-ray may not discriminate between cancer and tuberculosis, as neither the presence or absence of dyspnea does.

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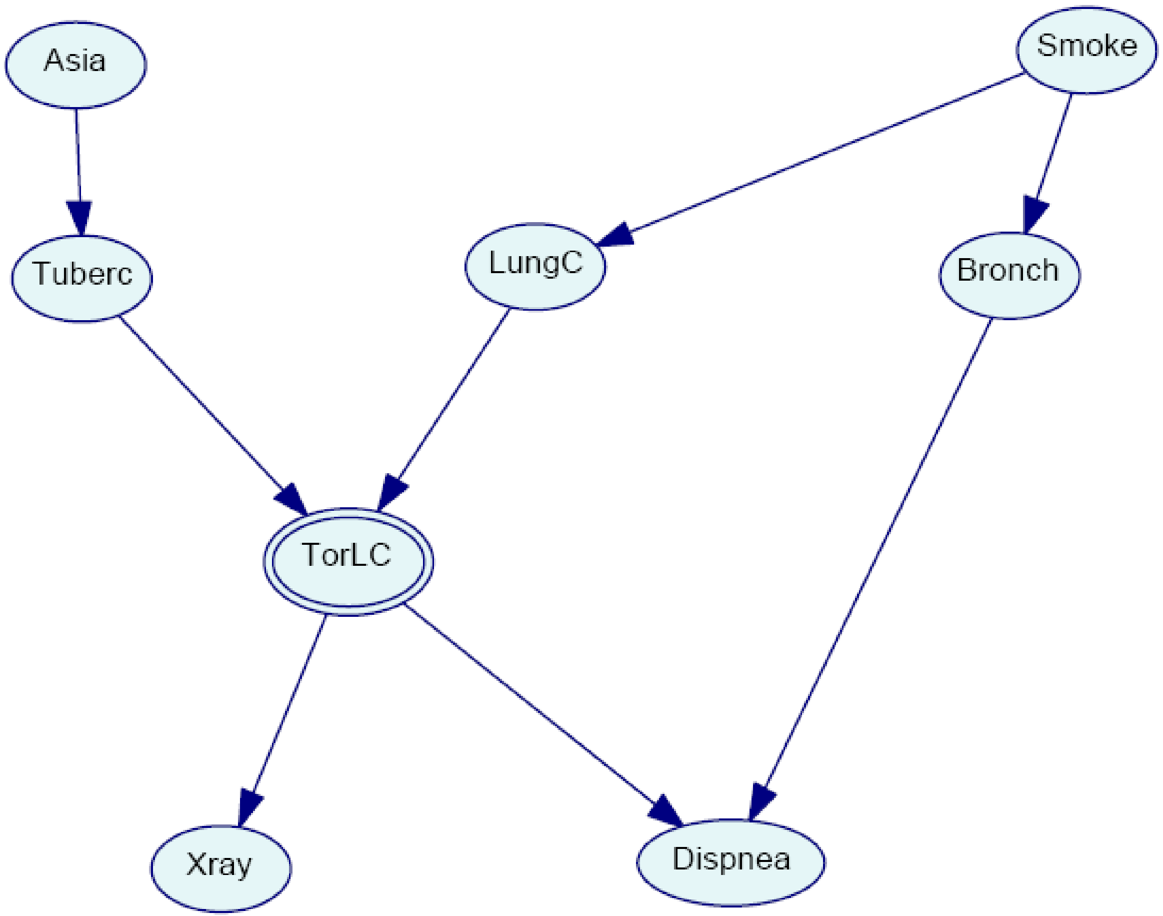
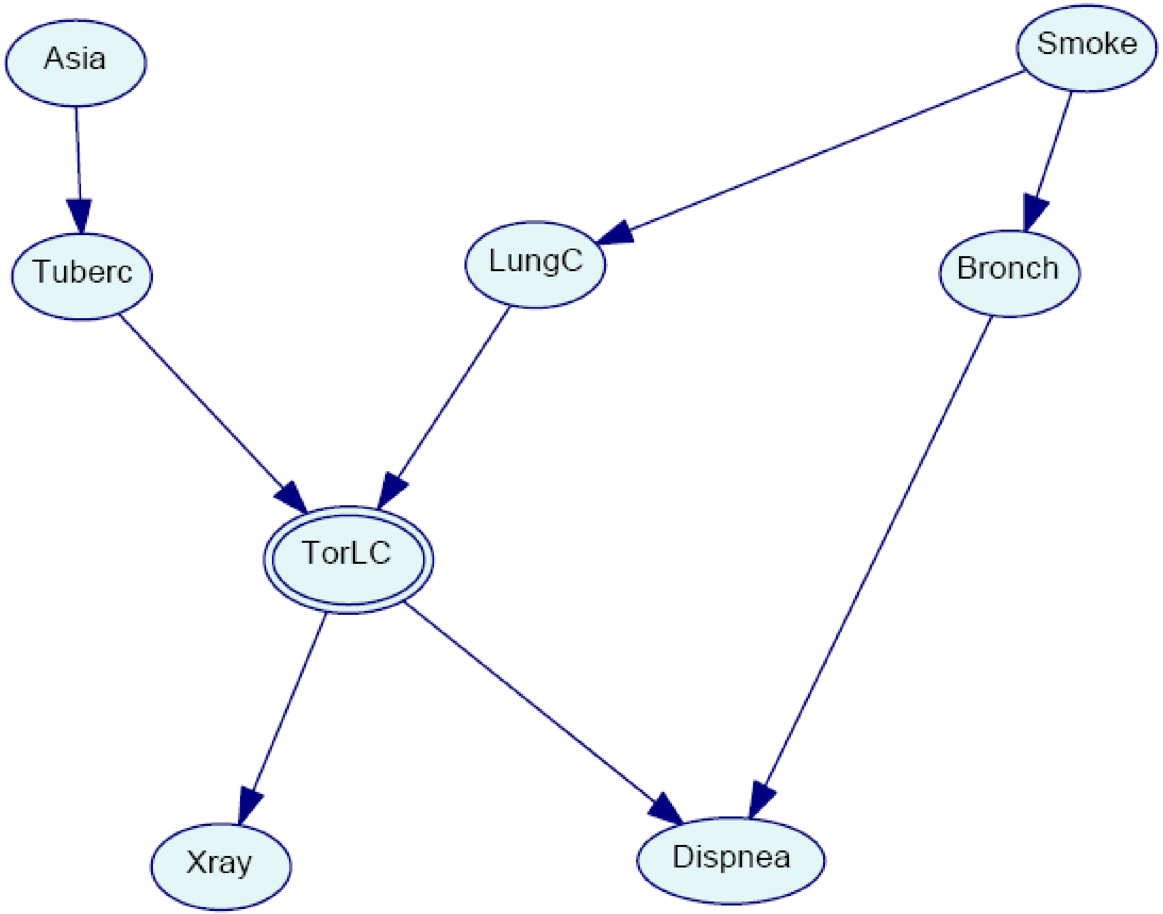


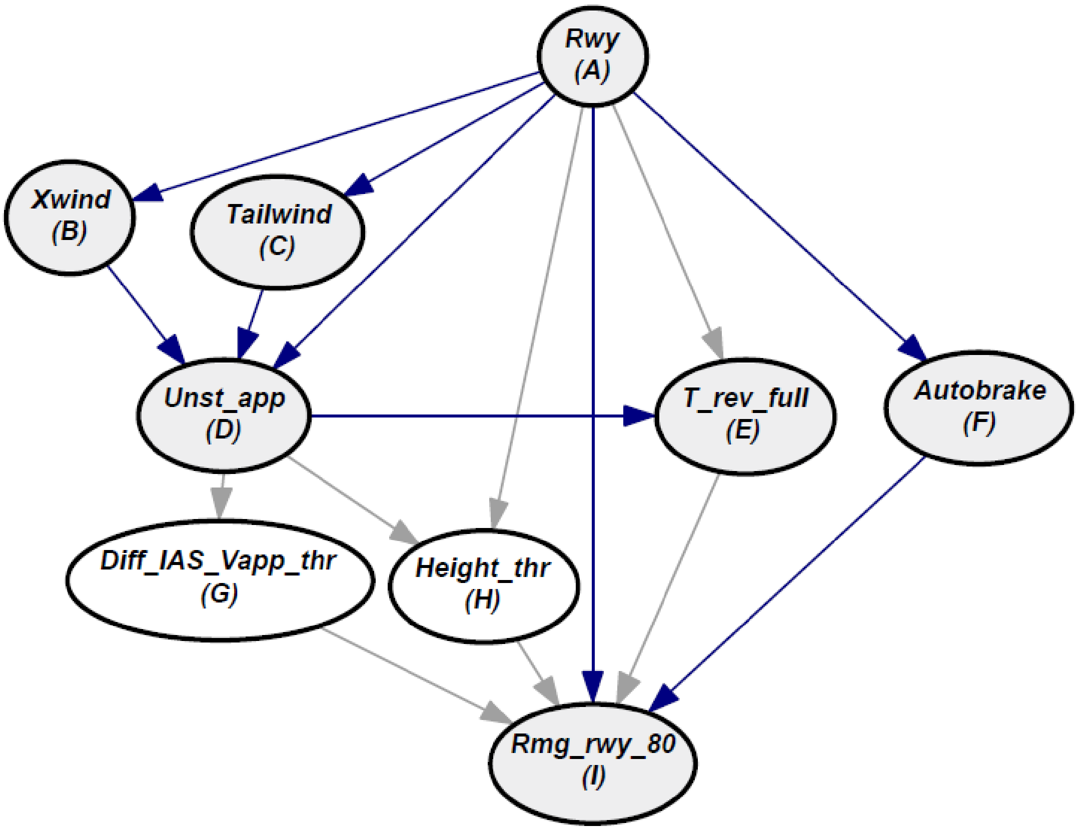
Provide the model

Probabilistic diagrams. Asia



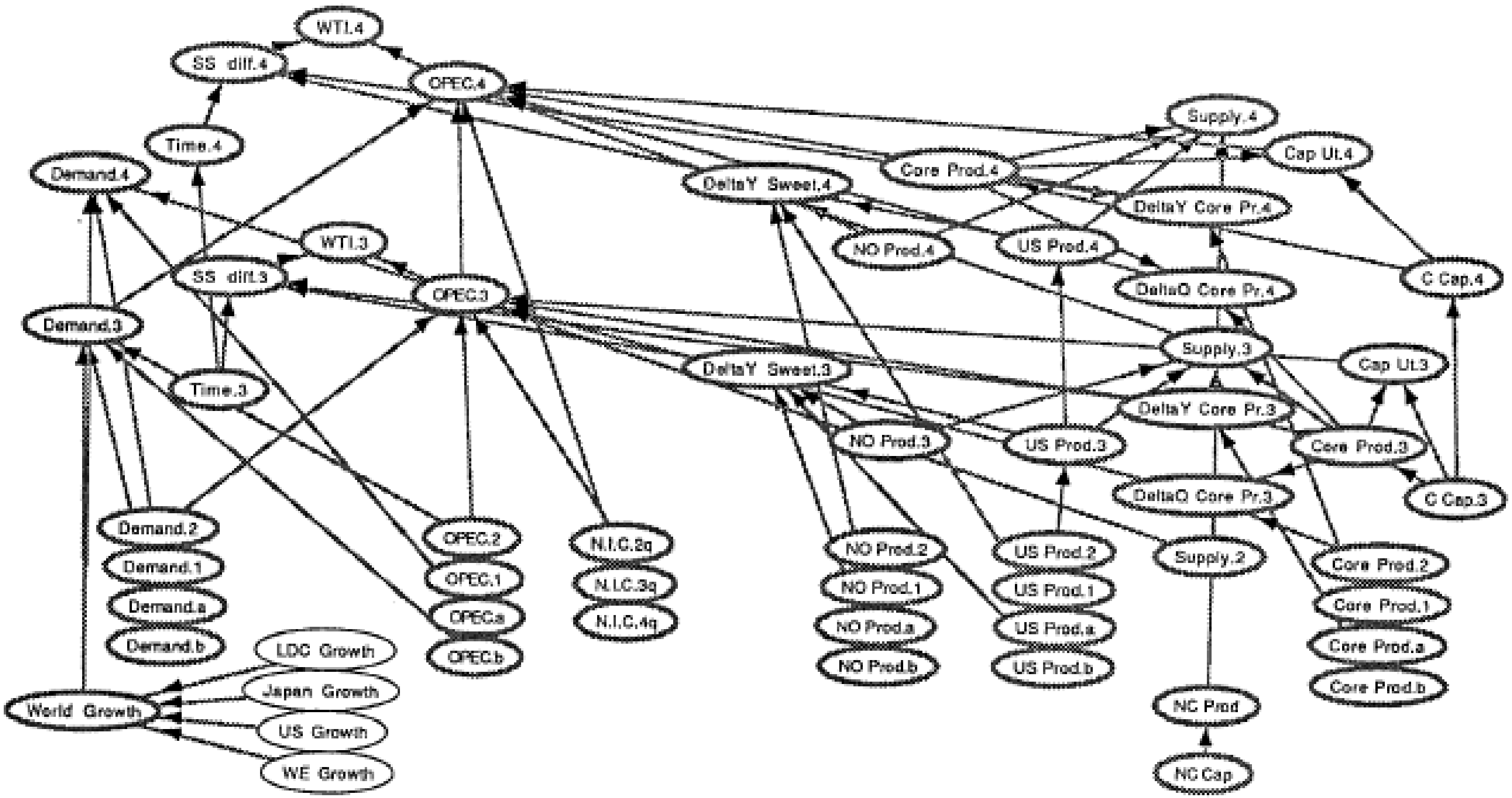
P(A,T,S,L,B,O,X,D) = P(A)P(T|A)P(S)P(L|S)P(B|S)P(0|T,L)P(X|O)P(D|O,B)

Runway excursions at airports



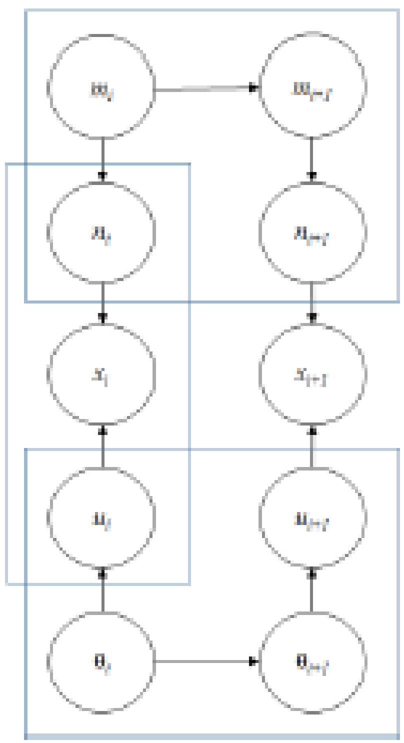
Build the probabilistic model

Model for oil prices



Forecasting aviation incidents

- For type k incident



$$
\begin{cases}
x_i|\lambda_i, n_i \sim Po(\lambda_i n_i) \\
u_i = log\lambda_i \\
\begin{cases}
\begin{cases}
u_i = F_i\theta_i + v_i, v_i \sim N(0, V_i) \\
\theta_i = G_i\theta_{i-1} + w_i, w_i \sim N(0, W_i)
\end{cases} \\
\theta_0 \sim N(\mu_0, W_0)
\end{cases} \\
\begin{cases}
\begin{cases}
n_i = H_i m_i + z_i, z_i \sim N(0, \Sigma_i) \\
m_i = J_i m_{i-1} + \xi_i, \xi_i \sim N(0, S_i)
\end{cases} \\
m_0 \sim N(\eta_0, S_0)
\end{cases}
\end{cases}
$$